

# Exploring stability and other fundamentals in a simple economy model

*On Sustainability, Stability, Utility Functions and Maximum Profit in Real Business Cycles*

Marcel R. de la Fonteyjne

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## Abstract

In this essay I will show you, from a theoretical point of view, the connection between sustainability, stability, welfare and maximum profit in real business cycles. In a simple one sector Cobb-Douglas economy I investigate the nature of economics. Attention is given to system stability, sustainability, unemployment, maximum profit conditions and consumer utility functions in real business cycles. As a result, I conclude that in general the goals of the actors in the economic field are not serving the mutual interest of society as a whole. E.g. maximum consumer utility does not coincide with maximum producer profit unless their profit is zero when we require also full employment in equilibrium. Furthermore, we show that even the simplest economy model can be intrinsically unstable. An odd result worthwhile further investigation.

**Keywords:** sustainability, stability, utility function, maximum profit, real business cycle

**JEL Classification** A10 · C00 · E20

## Introduction

As an advisor on sustainable energy and resource use I was wondering how sustainability and economic stability were incorporated in economy models. In order to investigate this item, I started to examine to date Dynamic Stochastic General Equilibrium (DSGE) models. After a short while I realized that these models were already that complex and have so many parameters allowing them to fit every past event, that they are capable in 'predicting' almost every event in the past, but have serious trouble in predicting the future, especially when these events involve new phenomena.

That's why I simplified 'my' economy to the maximum possible. This downsized economy does not have foreign influence and has no governmental organization. This task is directly performed by consumers and or producers. Because I assume that money is travelling with the speed of light I only need 1 US\$ to keep this economy going. Our economy is completely transparent and we do not need a banking section because consumers and producers will take care for it themselves. This will leave us with households and companies who can act as a consumer, saver, producer or as an investor. So far some boundary conditions.

At first I will have a closer look at each of our actors and their actions at the risk of being annoying, because also e.g. Smith, Marx and Keynes paid a lot of attention to this subject, but I will do it in a slightly different and shorter way. Especially In *The General Theory of Employment, Interest and Money* (Keynes, 1936) a few chapters are dedicated to income, saving and employment. Another reason is that I want to convince myself that every step is logic and fully consistent and has no flaws from a fundamental point of view.

Secondly I will show you an example and analyze some relations between sustainability, stability, employment, maximum profit conditions and consumer utility functions for a simple economy, followed by some conclusions.

### **Consumers, savers, producers and investors.**

Because I like you to join me on my examining trip I like to continue my story by writing 'we' as referring to you and me.

We will start with the construction of our simple economy.

We assume that consumers have the possibility to decide to buy and consume the amount  $C$  they desire within the limits of their income. Our economy is transparent and customers tell producers what they like and producers produce exactly what is needed and the level of inventory can be considered as minimal and not changing.

Producers on the other hand can decide which amount they will invest and are going to buy from other capital goods producers limited by the amount saved by the savors.

Because the total of these two purchases have to be equal to the total amount of production  $W$ , we can write:

$$W=C+I \quad (\text{eq. 1})$$

This is also equal to the amount to be paid to the producers.

The producers have to pay the workers a wage  $w$  for the number of labor units  $L$  and they have to pay for the use of capital  $K$ . Direct or indirect these payments will end up with the consumers income  $Y$  i.e.

$$Y = wL + (\delta + r)K \quad (\text{eq. 2})$$

in which  $\delta$  is depreciation of capital  $K$  and  $r$  is interest on capital use.

As consumers can decide to spend  $C$ , they will save  $S$  as the remaining part of  $Y$ .

$$Y = C + S \quad (\text{eq. 3})$$

If we look at the production side we assume (for now) the production to be dependent on  $L$  and  $K$ :

$$W = F(L,K) \quad (\text{eq. 4})$$

This production  $W$  has the value  $Y$

$$W = Y \quad (\text{eq. 5})$$

As a result, we conclude that the investments I will equal the savings S.

$$I = S \quad (\text{eq. 6})$$

Moreover, we can consider capital as accumulated labor combined with energy E (from the sun) and resources R (from mother earth). We assume this energy E and resources R are available for free and it becomes valuable once we add labor to exploit those resources. Action to preserve the environment can be thought as part of the consumption C once we agree upon this to do so and for the sake of simplicity we consider knowledge (human capital, research, entrepreneurial spirit, etc.) as a part of capital K, although this is not a very satisfying reasoning if you try to explain the productivity growth, but for our purpose it is sufficient.

Now we realize of course that the driving force on earth is energy from the sun. No sun, no plants, no fossil energy, no acceptable environment for human creatures and probably no life at all. Fortunately, the sun is going to stay there for a while, which we normally like to refer to as an infinitely sustainable energy source.

The question arises what is driving our economy. It probably starts by the fact that we want 'something' and our first action will be to work to put thing in place and for that of course a lot conditions have to be fulfilled. E.g. if one has nothing and wants to stay alive you could begin to start picking cranberries in the wood.

From the above we can conclude that economy starts with labor and is driven by labor and is in principle driven by labor only (Hazlitt, 1946) although under boundary conditions.

### **A specific case**

We choose the production function F to be a homogeneous Cobb-Douglas function only for demonstration and convenience, because we can derive formulas in explicit and simple form. The philosophy stays the same if we choose an arbitrary function.

$$F(L,K) = pL^{(1-\alpha)}K^\alpha \quad (\text{eq. 7})$$

In which p and  $\alpha$  are parameters with respect to our economy. It is also possible to incorporate human capital and other factors to clarify Solow Residual, but for now we take this simple expression (Donselaar, 2011).

We consider the equations:

$$Y = C + I \quad (\text{eq. 8})$$

$$Y = pL^{(1-\alpha)}K^\alpha \quad (\text{eq. 9})$$

$$\dot{K} = I - \delta K \quad (\text{eq. 10})$$

Scaling with L will result in the known equations:

$$y = c + i \quad (\text{eq. 11})$$

$$y = pk^\alpha \quad (\text{eq. 12})$$

$$\dot{k} = i - \delta k \quad (\text{eq. 13})$$

with the labor productivity  $y = Y/L$ , the capital to labor ratio  $k = K/L$ , the consumption to labor ratio  $c = C/L$  and the investment to labor ratio  $i = I/L$ .

If we now choose  $c = c_1 y$ , where  $c_1$  is the consumer part of income  $y$ , then we can solve the equilibrium solution of eq. 11-13 for  $k$  and  $y$  at every consumer's choice  $c_1$ . In fact,  $c_1$  is determined by  $c$  and  $i$  and so by consumers and producers. The equilibrium solution is:

$$k_{c_1} = \left( \frac{p(1-c_1)}{\delta} \right)^{\frac{1}{1-\alpha}} \quad (\text{eq. 14})$$

$$y_{c_1} = p \left( \frac{p(1-c_1)}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 15})$$

$$c_{c_1} = c_1 p \left( \frac{p(1-c_1)}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 16})$$

$$\left( \frac{k}{y} \right)_{c_1} = \frac{(1-c_1)}{\delta} \quad (\text{eq. 17})$$

Maximizing the utility function  $u = c$  (Kydland et al., 1982) results in

$$c_{1\_opt} = 1 - \alpha \quad (\text{eq. 18})$$

$$k_{opt} = \left( \frac{p\alpha}{\delta} \right)^{\frac{1}{1-\alpha}} \quad (\text{eq. 19})$$

$$y_{opt} = p \left( \frac{p\alpha}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 20})$$

$$c_{opt} = (1 - \alpha) p \left( \frac{p\alpha}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 21})$$

$$\left( \frac{k}{y} \right)_{c_{1\_opt}} = \frac{\alpha}{\delta} \quad (\text{eq. 22})$$

For a graph of  $c$ ,  $k$ ,  $y$  as a function of  $c_1$  see fig. 1 with  $\alpha = 0.3022$  and  $\delta = .079$  (7.9 %/year),  $w = 55.7$  ( $10^3$  €/year/L) and  $p = 18.2$  arbitrary chosen, which is close to the Dutch economy in 2010.

Notice that by putting these equations per labor unit will force capital  $k$  to be used to its full capacity to generate  $y$  and the part not used for consumption is invested.

By choosing  $c = c_1 y$  we introduce the consumers and producers behavior with respect to the dynamics of the system.

If we rewrite eq. 11-13 this results in:

$$\dot{k} = pk^\alpha - \delta k - c \quad (\text{eq. 23})$$

Again we choose  $c = c_1 y$  and linearizing around  $k_{c_1}$  gives

$$\dot{k} = \frac{\alpha}{\left( \frac{k}{y} \right)_{c_1}} k(1 - c_1) - \delta k \quad (\text{eq. 24})$$

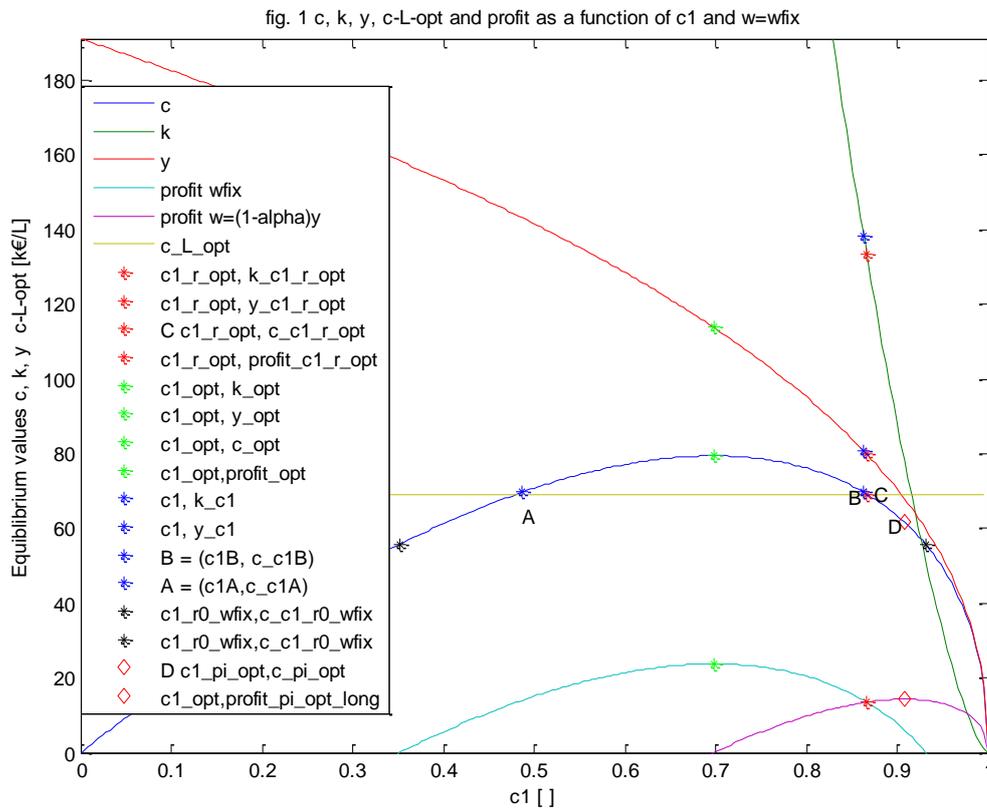
The eigen value of this equation is  $\lambda$

$$\lambda = \frac{\alpha}{\left(\frac{k}{y}\right)_{c_1}} (1 - c_1) - \delta = \delta(\alpha - 1) < 0 \quad (\text{eq. 25})$$

which holds for  $\forall c_1 \in [0,1)$

This means that this system is stable and will converge towards the equilibrium around  $c_1$  starting from initial condition  $\forall k_0 > 0$ . The time constant  $\tau$  is

$$\tau = \left| \frac{1}{\lambda} \right| = \frac{1}{\delta(1-\alpha)} \quad (\text{eq. 26})$$



In our example with  $\delta = 0.079$  and  $\alpha = 0.3022$  the time constant is  $\tau = 18.1$  year. Notice that it takes several decades to reach a new capital equilibrium in a smooth way.

Some special points:

At  $c_1 = 0$  also  $c=0$  and all income  $y$  will be spent on replacement investments, which is equal to depreciation, so  $y = i = \delta k$  and  $k = \left(\frac{p}{\delta}\right)^{\frac{1}{(1-\alpha)}}$  and at  $c_1 = 1$  our economy has vanished, i.e.  $c = y = k = 0$  as can be seen in fig. 1.

If the consumers want to maximize their utility  $u$  defined by  $u=c$  then they have to choose

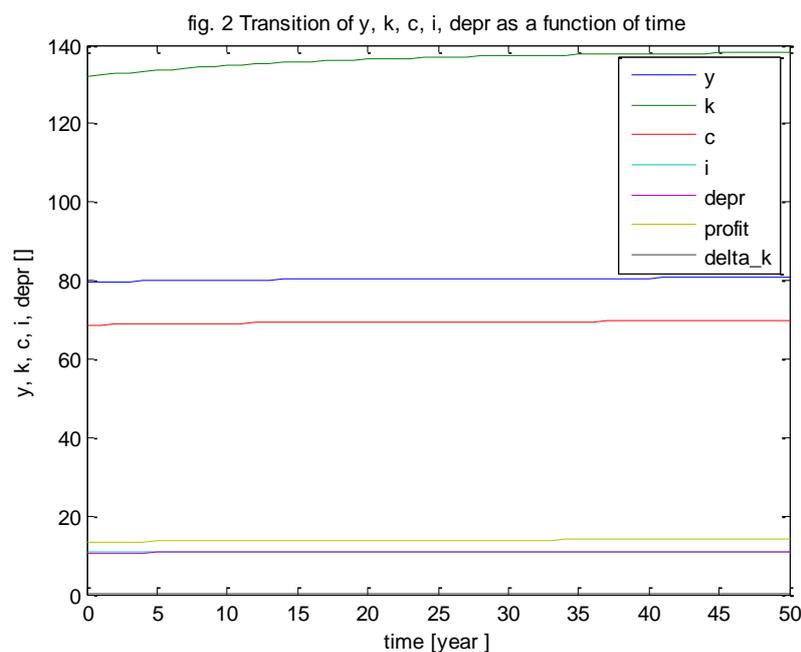
$$c_1 = c_{1\_opt} = 1 - \alpha$$

Now we arrive at a very interesting point. We have to question ourselves, can we really freely choose  $c_1$  to achieve this optimum utility value or does it come at the expense of something.

Choosing  $c_1 < c_{1\_opt} = 1 - \alpha$  will lead to a lower utility value, but higher income  $y$  if we pass the optimum value and because  $k$  is higher, so is the depreciation. Now capital  $K$  is generated in the end by labor  $L$ , sun energy  $E$  and earth resources  $R$ . This means that we use more resources and gain less. So this is a bad choice.

If we choose  $c_1 > c_{1\_opt} = 1 - \alpha$  this will also lead to a lower utility value but now  $y$  will be lower if we pass the optimum value and the depreciation will also be lower. Again, capital  $K$  is generated in the end by labor  $L$ , sun energy  $E$  and earth resources  $R$ . This means that we use less resources and gain less. So this might be a negotiable choice where we have to balance sustainable resource use and utility gain. Keep in mind that we only considered the resources use with respect to capital use. Total environmental impact is related to  $y$  and not only to  $\delta k$ .

Until so far we calculated the equilibrium condition. Now let's have a look on the dynamics in our first order non-linear system. We already know that our system is stable in the first order approximation. We start e.g. in an equilibrium with starting condition  $k_0$  and  $c_1$ . If we choose a new value for  $c_1$  the system will move to its new equilibrium value following approximately an exponential function with time constant  $\tau$ . As an example we calculate for  $k_0 = 134.01$  and  $c_1 = .8810$  the new equilibrium value at  $c_1 = .8780$  to be  $k = 138.93$  (fig. 2).



The next exercise is to change the consumer's behavior  $c_1$ . Suppose  $c_1$  will increase. Initially there is no change in capital  $k$ , because capital takes time to build, which means that the initial production value  $y$  (full employment) stays the same. From  $y = c + i$  we calculate that  $i$  will initially decrease by the same amount as  $c$  increases. The complete process will result eventually in a lower  $c$  than the initial value. One could say that we consume ourselves towards a lower consumption level. The opposite operation holds as well and results in a contrary reasoning. In this case we 'save' ourselves towards a higher consumption level.

As an example we will choose  $c_1 = c_{\text{equi\_new}}$  in such a way that  $k$  will return to its original value  $k_0$ . We take  $c_{\text{equi\_new}} = 0.8810$ , which will do the job as expected.

Unfortunately, this behavior is not what is seen in real economy when we look at investments which tend to have the same character as consumption as a function of time and is also more volatile. In most cases investment and employment is strong positive correlated with consumption  $c$  and income  $y$ .

At this point we introduce labor to understand what this will mean for the number of labor units required.

We use equations 8-10 and

$$Y = wL + (\delta + r)K \quad (\text{eq. 27})$$

Eq. 27 divided by  $L$  result in

$$y = w + (\delta + r)k \quad (\text{eq. 28})$$

Maximizing with respect to producer profit yields the equations:

$$(1 - \alpha)y = w \quad (\text{eq. 29})$$

$$\alpha \frac{y}{k} = \delta + r \quad (\text{eq. 30})$$

For each  $c_1$  we can calculate a corresponding  $w$  and  $r$  for which maximum profit holds. Notice however, that maximum profit coincides with maximum utility at  $c_1$  if and only if  $c_1 = c_{1\_opt} = 1 - \alpha$

Resulting with  $\left(\frac{k}{y}\right)_{c_1} = \frac{\alpha}{\delta}$  (eq. 22) and eq. 30 in

$$r = 0 \quad (\text{eq. 31})$$

A remarkable result: maximum profit is zero for producers, a situation only acceptable in e.g. the old Russian situation with pure central planned economy, but not in a capitalistic economy (Marx, 1867). Perhaps there are other possible solutions, but they will probably turn out to be not feasible or not practical to realize. An escape of the problem might be a more holistic view on humanity, economics and resource use, where e.g. ownership of companies is more or less equally spread over the theoretical production factors labor, capital and human capital (Van Egmond, 2010). One thing is clear, only focusing on growth of GDP is not a very successful instrument in leading the way to sustainability and welfare, perhaps on the contrary (Meadows & Randers, 2004).

But there is another reason why it is better to choose  $c_1 > c_{1\_opt} = 1 - \alpha$  (we already rejected the possibility  $c_1 < c_{1\_opt} = 1 - \alpha$ ).

Suppose our economy has  $L$  labor units available. From eq. 8-10 and 27 we derive a formula for  $L_{\text{opt}_c1}$  to maximize profit at  $c_1$  and so at constant  $K_{c_1} = Lk_{c_1}$  by differentiating with respect to  $L$ .

This results in

$$L_{opt\_c1} = \left( \frac{p(1-\alpha)}{w} \right)^{\frac{1}{\alpha}} L k_{c1} \quad (\text{eq. 32})$$

$L_{opt\_c1}$  is the value of L for which our system has a stable solution at  $c1$  and maximum profit conditions.

If we require no unemployment  $L_{opt\_c1}$  must equal L,  $L_{opt\_c1} = L$ .

From eq. 32 we can solve the wage w

$$w_{L_{opt\_c1}} = p(1-\alpha)k_{c1}^{\alpha}$$

So for every  $c1$  there exists a w which maximize profit interest r on K.

And of course the inverse formulas hold as well. For w given we derive at maximum profit:

$$c_{1r_{opt}} = 1 - \frac{\delta}{p} \left( \frac{w}{(1-\alpha)p} \right)^{\frac{(1-\alpha)}{\alpha}} \quad (\text{eq. 33})$$

$$k_{c_{1r_{opt}}} = \left( \frac{w}{(1-\alpha)p} \right)^{\frac{1}{\alpha}} \quad (\text{eq. 34})$$

$$y_{c_{1r_{opt}}} = \frac{w}{(1-\alpha)} \quad (\text{eq. 35})$$

$$r_{c_{1r_{opt}}} = \frac{\alpha w}{(1-\alpha)} / \left( \frac{w}{(1-\alpha)p} \right)^{\frac{1}{\alpha}} - \delta \quad (\text{eq. 36})$$

In our example  $c_{1r_{opt}} = 0.8810$  and  $k_{c_{1r_{opt}}} = 134.01$  and we calculated p and  $\alpha$  from the initial values.  $L_{c1\_actual} = L_{opt\_c1} < L$  corresponding with a little unemployment the actual situation in 2010.

If we choose another consumer behavior e.g.  $c=c_0=\text{constant}$  than we can show that this will result in two solutions for  $c1$  for which the left point A at  $c_{1A}$  in fig. 1 is stable and the right one B at  $c_{1B}$  is unstable. As we discussed earlier the left point A is not a sustainable solution though possible and we therefore will only consider the unstable solution B. For  $c1 > c_{1A}$  the income y will decrease and become zero over time. For  $c1 < c_{1A}$  the income y and capital k will increase toward point A.

It is interesting to notice that by changing the consumers (and producers) behavior one can change the economy from growing to declining and we will examine how this can be achieved.

For that we combine the two consumer behavior strategies and define

$$c_1 y = c_0 + c_2 y \quad (\text{eq. 37})$$

After linearizing around equilibrium in the same way as eq. 22 this results in

$$\dot{k} = \frac{\alpha}{\left( \frac{k}{y} \right)_{c1}} k(1 - c_2) - \delta k \quad (\text{eq. 38})$$

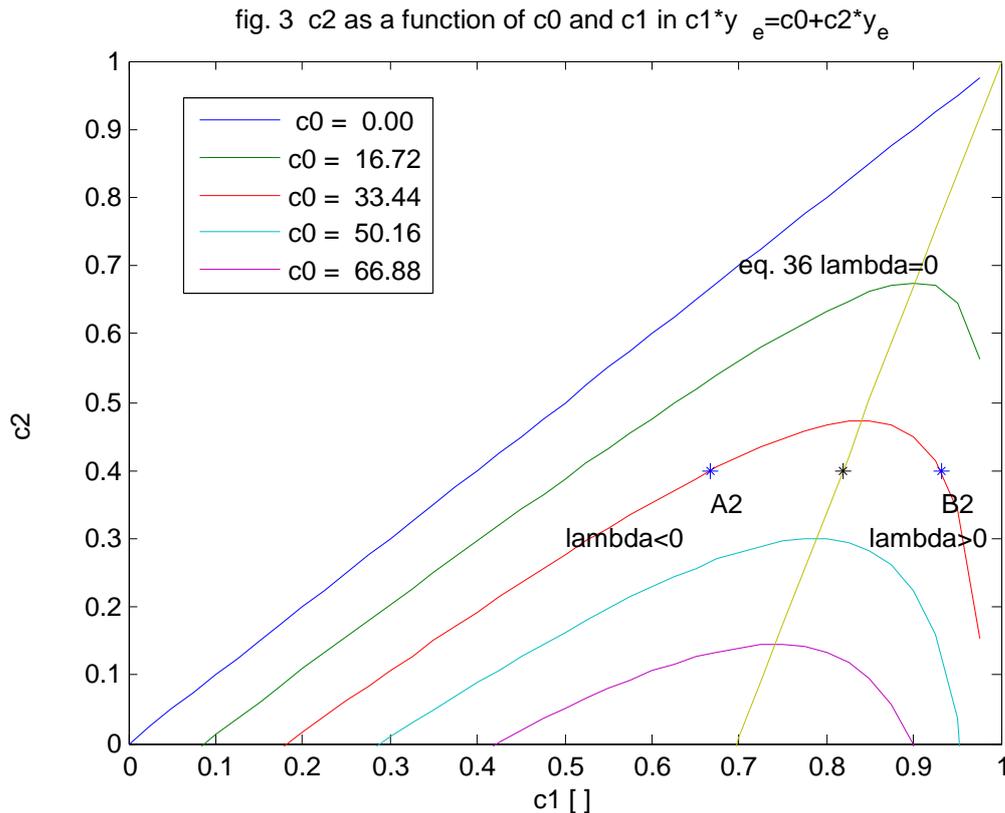
The eigen value for this system is

$$\lambda = \left( \frac{\alpha(1-c_2)}{(1-c_1)} - 1 \right) \delta \quad (\text{eq. 39})$$

The solution is stable for  $\lambda < 0$  which is the same as requiring

$$\alpha(1 - c_2) < (1 - c_1) \quad (\text{eq. 40})$$

The equation for which  $\lambda = 0$  is shown in fig. 3 and is connecting all maxima of equation 37.



To the left of this line we have a stable solution and to the right of this line we have an unstable solution and near this line the system is very slow responding system because  $\lambda = 0$  or close to zero.

Choose as an example e.g.  $c_2 = .4$  and  $c_0 = 33.44$  then there are three equilibrium points. Point B2 (fig.3) is unstable because it is to the right side of the line  $\lambda = 0$  with  $\lambda > 0$ ,  $c_1 = .9318$  and  $k_{B2} = 60.51$ . The second solution is point A2 with  $\lambda < 0$ ,  $c_1 = 0.6680$  and  $k_{A2} = 584.36$ , which is a stable solution. If the initial condition is  $k_0 > k_{B2}$  then the solution is A2 and if  $k_0 < k_{B2}$  then the solution is  $k = 0$  and  $y = 0$ , which is the third solution.

If  $c_0 = 33.44$  and  $c_2 > 0.472$  then only the trivial zero solution exists.

Notice that if  $k$  is moving towards zero and we require the investment to be bigger than zero then there is a moment in time that  $c = y$ , which means that  $c_1 = 1$  and  $y = \frac{c_0}{1-c_2}$ . From this moment on the consumer behavior changes to  $c = y$ .

Suppose we want to operate the economy in  $c_1$  and we were able to adapt consumer behavior  $c_0$  and  $c_2$  in such a way that  $\lambda = 0$  then we have to choose  $c_2 = 1 - \frac{(1-c_1)}{\alpha}$  and  $c_0 = (c_1 - c_2)y$ . The result is that point A2 and B2 will coincide. It is stable when the initial condition  $k_0 > k_{B2}$  and unstable when  $k_0 < k_{B2}$ .

### Investment as a function of profit

Again we start with eq. 11-13 and 25

$$y = c + i \quad (\text{eq. 41})$$

$$y = pk^\alpha \quad (\text{eq. 42})$$

$$\dot{k} = i - \delta k \quad (\text{eq. 43})$$

$$y = w + (\delta + r)k \quad (\text{eq. 44})$$

and add consumer and producer behavior

$$c = c_1 y \quad (\text{eq. 45})$$

$$i = (1 - d)rk + \delta k \quad (\text{eq. 46})$$

where  $d$  is dividend. From an investment point of view this is a reasonable assumption.

After substitution this yields

$$\dot{k} = (1 - d)rk = (1 - d)(y - w - \delta k) \quad (\text{eq. 47})$$

Which has an equivalent form as eq. 23 and after putting  $w = c_0 + c_2 y$  and linearizing, this result in the same formulas as eq. 38 - 40 and we can use the same diagram as in fig. 3.

If we choose  $w$  fixed  $w = c_0$ , we can solve

$$y - w - \delta k = 0 \quad (\text{eq. 48})$$

to find equilibrium values for  $k$ ,  $y$  and corresponding  $c$  for which the smaller values are unstable and higher values are a stable equilibrium.

If we choose  $w = (1 - \alpha)y$ , which is the maximum profit condition, then eq. 48 reduces to

$$\left(\frac{k}{y}\right)_{c_1} = \frac{\alpha}{\delta}, \text{ which is the same as eq. 22 with } c_1 = c_{1r_{\text{opt}}} \text{ and } r=0.$$

Notice that  $w = (1 - \alpha)y$  is also the condition for preserving  $\alpha$ .

### Maximizing profit under the condition of constant labor share.

Constant labor share means that  $w = (1-\alpha)y$ . Profit under this condition is

$$\pi = rk = y - w - \delta k = \alpha y - \delta k = \alpha p k^\alpha - \delta k \quad (\text{eq. 49})$$

differentiating with respect to  $k$  and putting to zero result in

$$\frac{\partial \pi}{\partial k} = \alpha^2 p k^\alpha - \delta = 0 \quad (\text{eq. 50})$$

with solution

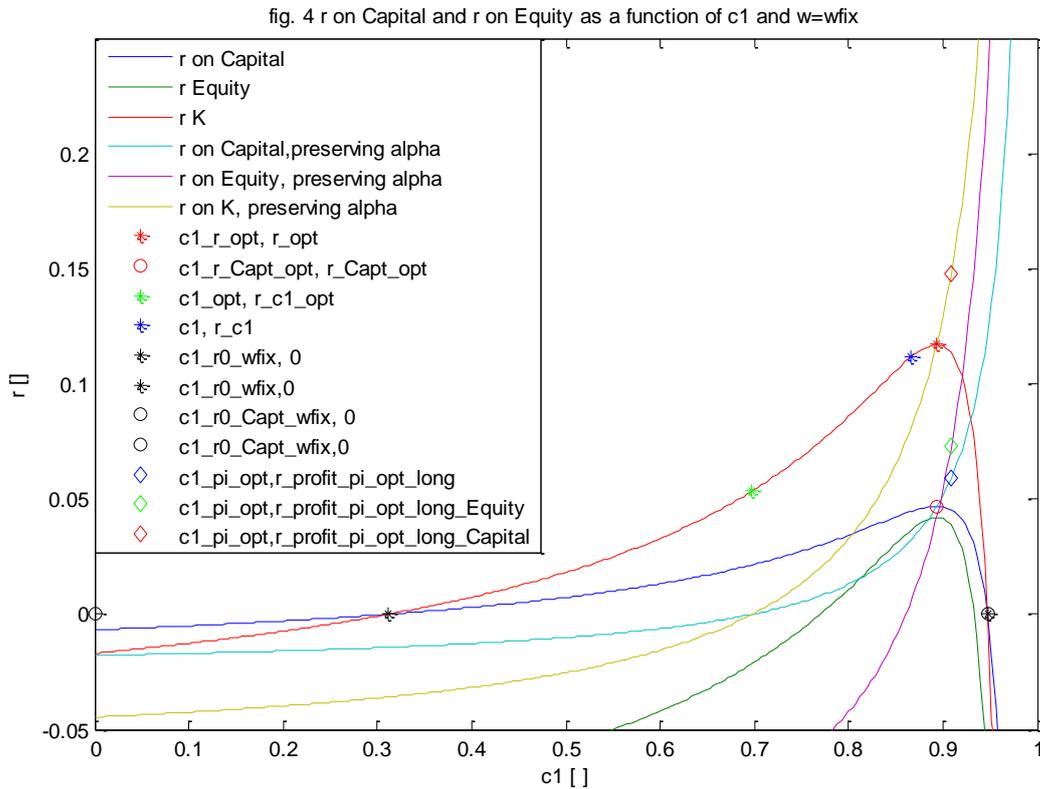
$$k_\pi = \left( \frac{\alpha^2 p}{\delta} \right)^{\frac{1}{1-\alpha}} \quad (\text{eq. 51})$$

$$c_{1\pi} = 1 - \alpha^2 \quad (\text{eq. 52})$$

$$y_\pi = p \left( \frac{\alpha^2 p}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 53})$$

$$c_\pi = (1 - \alpha^2) p \left( \frac{\alpha^2 p}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 54})$$

$$w_\pi = (1 - \alpha) p \left( \frac{\alpha^2 p}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{eq. 55})$$



If we compare the profit function with fixed  $w$  and the profit function with  $w = (1 - \alpha)y$  both as a function of  $c_1$  (see fig. 1), we see that the intersection lies exactly at  $c_{1r_{opt}}$  and corresponds with eq. 33-36.

Profitability on equity can be calculated from

$$r_{\text{equity}} = \frac{r_{\text{capital}} - r_{\text{liability}}(1 - p_{\text{equity}})}{p_{\text{equity}}} \quad (\text{eq. 56})$$

where  $p_{\text{equity}}$  is equity share of total firm capital and  $r_{\text{liability}}$  is interest on debt and

$$r_{\text{capital}} = r p_{\text{capital}} \quad (\text{eq. 57})$$

where  $p_{\text{capital}}$  is the share of K in firm Capital and  $r_{\text{capital}}$  profitability on firm Capital.

The curves for our example is shown in fig. 4 as well as the corresponding maxima of eq. 51 – 55.

## Sustainability

As a start to give you an idea of my intention with respect to sustainable production consider eq. 33 -36 where there is equilibrium for the economy with full employment. Suppose the population is fixed with L equivalent labor units available and that p is increasing over time, then  $c_1$  will go towards 1,  $c_1 \rightarrow 1$ ,  $k \rightarrow 0$ ,  $r \rightarrow \infty$  and

$$rk \rightarrow \frac{\alpha w}{(1-\alpha)} \quad (\text{eq. 58})$$

At this point c will move towards

$$c = c_1 y \rightarrow y = \frac{w}{(1-\alpha)} \quad (\text{eq. 59})$$

This means that if we would be satisfied by consuming the amount c, which is practically equal to y, the environmental burden with respect to capital needed will decrease towards zero,  $\delta K = \delta k L \rightarrow 0$ , when p increase under the assumption that L will not grow.

You might wonder if this result is also possible with increasing utility for consumers. For that we make the wage w dependent of p, e.g.:

$$w(p) = ap^\gamma \quad (\text{eq. 60})$$

Calculating c results in

$$c = c_1 y \rightarrow \frac{ap^\gamma}{(1-\alpha)} \quad (\text{eq. 61})$$

and

$$k \rightarrow \left( \frac{a}{(1-\alpha)} \right)^{\frac{1}{\alpha}} p^{\frac{\gamma-1}{\alpha}} \rightarrow 0 \text{ for } \gamma < 1 \quad (\text{eq. 62})$$

Now this gives the impression that this result is also possible with respect to C and K when the population will grow. In this case we define

$$L(p) = bp^\epsilon \quad (\text{eq. 63})$$

Calculating C results in

$$C = Lc_1 y \rightarrow \frac{abp^{\gamma+\epsilon}}{(1-\alpha)} \quad (\text{eq. 64})$$

and

$$K \rightarrow \left( \frac{a}{(1-\alpha)} \right)^{\frac{1}{\alpha}} b p^{\frac{(\gamma-1)}{\alpha} + \varepsilon} \rightarrow 0 \text{ for } \gamma + \alpha\varepsilon < 1 \quad (\text{eq. 65})$$

This means that we are able to control over time, as  $p$  increases, the environmental impact of capital use with respect to production, even when consumption  $C$  and population are increasing.

Time for some conclusions.

## Conclusions

Our system with  $c = c_1 y$ , starting from  $k = k_0$ , can be in equilibrium for every  $c_1$  or will move towards it.

Additionally when  $c_1 = c_{1_{\text{r}_{\text{opt}}}}$  there will be full employment,  $L_{\text{opt}_{c_1}} = L$ . And as a result:

If  $c_1 < c_{1_{\text{r}_{\text{opt}}}}$  then there is an upward pressure for more labor units.

If  $c_1 > c_{1_{\text{r}_{\text{opt}}}}$  then there is a downward pressure for less labor units.

If the fulfillment of full employment is desired then the choice of  $c_{1_{\text{r}_{\text{opt}}}}$  is predetermined if  $w$  is given, because it is preconditioned by our economy, i.e. by the actual status of technology and capital-income ratio, which is equivalent by saying to the parameters  $p$  and  $\alpha$ .

Dependent on the actual value of  $k = k_{\text{act}}$ , a Keynesian stimulus is useful or counterproductive in the economy described above.

In the case we use consumer behavior  $c = c_2 y + c_0$  in our economy model, dependent on the parameter  $c_2$  and  $c_0$  this can result in a stable or unstable equilibrium.

The maximizing utility strategy of consumers is irreconcilable with the maximum profit strategy of producers when full employment in equilibrium is required as well and is also different from sustainable resource use. It would be interesting to examine whether relationships and behavior of all actors in society can be regulated, guided or defined in such a way that interests become mutual.

I recommend to investigate this mechanism as to my opinion it will shed a different light on the behavior of real business cycles and how to avoid or deal with them. Special attention should be given to the differences with Smith, Marx, Keynes, Hazlitt, Lucas and Kydland & Prescott, to name a few I admire for their theoretical work.

In particular, I prefer to extend this research with monetary influence in relation to employment, consumer and producer credit facilities, public debt, foreign debt, economic stability, banking and central banking policies and the origination and avoidance of e.g. housing-bubbles.

We already mentioned the fact that in real economy fluctuations in investment  $I$  is more or less strongly positive correlated with fluctuation in income  $Y$ . And even in this case there is a stable and unstable solution. It is very promising to examine the consequences of such behavior in the same way as we did in this essay to understand the origination of real business cycles. In this case it seems useful to discriminate between consumer goods producers and capital goods

producers because, the interaction between personnel exchange and real business cycles might have some consequences as well, but we leave that for a next paper.

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